FAR BEYOND

MAT122

Power Rule and e^x Derivative

Stony Brook University

Intro

So far, it has been shown that a function has a derivative that tells about the rate of change between the independent variable "x" and the dependent variable "y".

The derivative can be measured using the slope of the tangent line at some value x.

Until now, that slope or $f'(x)$ has been estimated by finding the slope of nearby secant lines and with the definition of the derivative.

Now, a new way to differentiate is introduced!

Power Rule

use <u>only</u> with **power** functions

$$\frac{d}{dx}ax^n = n\,a\,x^{n-1}$$

ex. $f(x) = 4x^3$

$$\frac{d}{dx}f(x) = f'(x)$$

$= \boxed{12x^2}$

Power Rule (cont'd)

From here forward, notations will be interchanged at random.

ex. $(x^3)'$

$= \boxed{3x^2}$

ex. $\dfrac{d}{dx}5$

$= \boxed{0}$

$$(ax^n)' = \frac{d}{dx}ax^n = nax^{n-1}$$

ex. $\boxed{\dfrac{d}{dx}x = 1}$

$$\frac{d}{dx}a = 0$$

where a is a constant

$= \boxed{1}$

ex. $(2x)'$

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot \left(\frac{d}{dx}f(x)\right)$$

where c is a constant

$= \boxed{2}$

Power Rule with multiple terms

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

$$\frac{d}{dx}ax^n = nax^{n-1}$$

$$\frac{d}{dx}x = 1$$

$$\frac{d}{dx}c = 0$$

ex. $\frac{d}{dx}(x^4 - 6x^2 + 4)$

$= \boxed{4x^3 - 12x}$

Do: $\frac{d}{dx}(x^8 + 12x^6 - 4x^4 - 6x + \pi)$

Power Rule Application

$$(ax^n)' = \frac{d}{dx} ax^n = nax^{n-1}$$

ex. The revenue (in dollars) from producing q units of a product is given by $R(q) = 1000q - 3q^2$.

Find $R(125)$. Give units and interpret answer.

Find $R'(125)$. Give units and interpret answer.

Power Rule with Manipulations

Sometimes, a function is not given in power format.

$$\left(\frac{3x^{10}}{5}\right)' = \boxed{6x^9}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \boxed{-\frac{1}{x^2}}$$

$$\frac{1}{b^x} = b^{-x} \qquad b^{-x} = \frac{1}{b^x}$$

$$\frac{d}{dx}\sqrt{x} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\sqrt[n]{b} = b^{1/n}$$

$$\sqrt[3]{x^2}\,' = \boxed{\frac{2}{3\sqrt[3]{x}}}$$

$$b^x b^y = b^{x+y}$$

Higher Order Derivatives

ex. if $f(x) = -9x^2 + 7x$ find $f'(x)$, $f''(x)$, $f'''(x)$

Derivative of e^x

$$\frac{d}{dx}e^x = e^x$$

$$\text{if } f(x) = e^x - 2x^2 \quad \text{find} \quad f'(x),\ f''(x),\ f'''(x)$$